University of California, Berkeley
Physics 110B, Spring 2004 (*Strovink*)

ANSWERS TO ASSIGNED EXERCISES 1-72

1. (d.)

$$\lambda = \tfrac{1}{2}B_0xy$$

2.

$$4\pi\epsilon_0\vec{E} = \frac{q}{r^2}\hat{r}$$
$$\rho = q\delta^3(\vec{r})$$
$$\vec{B} = 0$$
$$\vec{J} = 0$$

3.

$$4\pi\epsilon_0 V' = \frac{q}{r}$$
$$\vec{A}' = 0$$

5.
It is always possible to find a gauge in which $V' = 0$, by using $\lambda = \int_0^t V(t')\,dt'$. But, in general, a gauge cannot be found in which $\vec{A}' = 0$, because that would force $\vec{B} = 0$.

6.
(b.) The constant of proportionality is 1/3! .
(c.)

$$\det A = \frac{1}{4!}\epsilon_{ijkl}A_{im}A_{jn}A_{kp}A_{lq}\epsilon_{mnpq}$$

10.
(a.) No (the interval is spacelike).
(b.) Yes.

$$|\vec{r}\,''_A - \vec{r}\,''_B| = \sqrt{2}$$

(c.) No (the interval is spacelike).
(d.) No (the interval is timelike).
(e.) Yes.

$$c|t''_E - t''_D| = \sqrt{7}$$

11. Λ is symmetric, and its independent elements are

$$\begin{pmatrix} \gamma & -\gamma\beta n_1 & -\gamma\beta n_2 & 0 \\ & \gamma n_1^2 + n_2^2 & (\gamma-1)n_1n_2 & 0 \\ & & \gamma n_2^2 + n_1^2 & 0 \\ & & & 1 \end{pmatrix}$$

12. The answers are the same.

15.

$$\eta_{\max} = 10.34$$
$$\beta_{\max} = 1 - (2.09\times 10^{-9})$$
$$x_{\max} = 2.84\times 10^{20}\text{ m} = 29,900\text{ light yr}$$
$$\Delta t = 1.89\times 10^{12}\text{ sec} = 59,850\text{ yr}$$

16. (b.)

$$\text{fraction decaying} = 1 - \exp\left(-L/\gamma_0\beta_0 c\tau\right)$$

17.
(a.)

$$L = \gamma\beta c\tau = \tfrac{4}{3}c\tau$$

(b.)

$$\psi = \cos^{-1}(2\beta^2 - 1) = 73.74^\circ$$

19. (b.) The dimensions of $d\sigma$ are Joules^{-2}.

24.
(a.)

$$I = 2ne\beta_0 cA$$

(b.)

$$\tfrac{1}{\mu_0}\vec{B} = \hat{\phi}\,\frac{ne\beta_0 cA}{\pi s}$$

(c.)

$$n'_+ = n/\gamma_0$$

(d.)

$$n'_- = n\gamma_0(1+\beta_0^2)$$

(e.)

$$c\epsilon_0\vec{E}' = -\hat{s}\,\gamma_0\beta_0\frac{ne\beta_0 cA}{\pi s}$$

26.
(a.)

$$F_{\mu\nu}F^{\mu\nu} = -\tfrac{2}{c^2}(|\vec{E}|^2 - |c\vec{B}|^2)$$

(b.)

$$F^{\mu\nu}G_{\mu\nu} = -\tfrac{4}{c}\vec{E}\cdot\vec{B}$$

(c.)

$$\vec{E} \perp \vec{B} \ \text{ and } \ |\vec{E}| > |c\vec{B}|$$

28.
(a.)

$$\sinh\eta = \frac{F_0}{mc}\,t$$

(b.)

$$t_1 = \frac{mc}{F_0}$$

29.
(c.) x does not increase linearly with t.

30.
(c.) z does increase linearly with t.

32.

$$\begin{aligned}
\vec{E}(\vec{r},t) &= \hat{z}\,\frac{\mu_0 q_0 c^3 t}{2\pi[(ct)^2 - s^2]^{3/2}} \quad \text{for } t > s/c \\
&= 0 \ \text{ otherwise} \\
c\vec{B}(\vec{r},t) &= -\hat{\phi}\,\frac{\mu_0 q_0 c^2 s}{2\pi[(ct)^2 - s^2]^{3/2}} \quad \text{for } t > s/c \\
&= 0 \ \text{ otherwise}
\end{aligned}$$

35.
(b.)

$$\vec{S} \to \frac{q^2 c}{16\pi^2\epsilon_0}\,\frac{\hat{z}}{b^2 z^2} \quad \text{as } \ \gamma \to \infty$$

(c.) No.

37.
(a.)

$$4\pi\epsilon_0 V(\vec{r}) \cong \frac{4ak}{\pi r}$$

(b.)

$$4\pi\epsilon_0 V(\vec{r}) \cong \frac{2a^2 k\cos\theta}{\pi r^2}$$

(c.)

$$4\pi\epsilon_0 V(\vec{r}) \cong -\frac{2a^3 k}{\pi^2 r^3}(3\cos^2\theta - 1)$$

38.
(a.)

$$\frac{\epsilon_0}{q}V(\vec{r}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y_{lm}(\theta,\phi)Y^*_{lm}(\theta',\phi')\,r'^l}{(2l+1)r^{l+1}}$$

(b.)

$$\begin{aligned}
&\frac{4\pi\epsilon_0}{q}V(\vec{r}) \cong \frac{1}{r} \\
&+ \frac{r'}{r^2}\big(\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi')\big) \\
&+ \frac{r'^2}{r^3}\Big((\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2})(\tfrac{3}{2}\cos^2\theta' - \tfrac{1}{2}) \\
&+ 3\sin\theta\sin\theta'\cos\theta\cos\theta'\cos(\phi-\phi') \\
&+ \tfrac{3}{4}\sin^2\theta\sin^2\theta'\cos\big(2(\phi-\phi')\big)\Big)
\end{aligned}$$

39.
Place charges $+1$, -4, $+6$, -4, $+1$ at $z = 2$, 1, 0, -1, and -2, respectively.

42.

$$f(\theta,\phi) \propto \cos^2\theta\sin^2\theta$$

43.
(a.) q_{22} and $q_{2\,-2}$ do not vanish and have equal relative weight.
(b.) E-type (TM) radiation of types $E2_2$ and $E2_{-2}$ are emitted with equal relative weight.
(c.)

$$f(\theta,\phi) \propto \sin^2\theta\,(1 - \sin^2\theta\cos^2 2\phi)$$

vanishes in six directions: $\theta = 0$, $\theta = \pi$, and at $(\theta = \frac{\pi}{2}; \phi = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2})$.

47.
(a.)

$$\omega' = \frac{2\pi\beta c}{\Delta z/\gamma} \quad \Big(\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}\Big)$$

(b.)

$$\omega = \frac{\omega'}{\gamma(1-\beta)}$$

(c.)

$$\begin{aligned}\lambda &= \Delta z\,\frac{1-\beta}{\beta}\\ &= \frac{\Delta z}{\gamma^2\beta(1+\beta)}\\ &\to \frac{\Delta z}{2\gamma^2}\ \text{ as } \beta\to 1\end{aligned}$$

(d.)

$$\begin{aligned}E &= \gamma\, m_e c^2 \approx \sqrt{\frac{\Delta z}{2\lambda}}\, m_e c^2\\ &= 1.616\ \text{GeV}\end{aligned}$$

51.
(b.) The state of polarization is RH (LH) elliptical if $\mathrm{Im}\,\beta < 0$ ($\mathrm{Im}\,\beta > 0$).
(c.)

$$\begin{aligned}\sqrt{1+|\beta|^2}\,\vec{J}_1 &= \begin{pmatrix}1\\ \beta\end{pmatrix}\\ &= -\mathrm{Im}\,\beta\begin{pmatrix}1\\ -i\end{pmatrix} + \begin{pmatrix}1+\mathrm{Im}\,\beta\\ \mathrm{Re}\,\beta\end{pmatrix}\\ &= \mathrm{Im}\,\beta\begin{pmatrix}1\\ i\end{pmatrix} + \begin{pmatrix}1-\mathrm{Im}\,\beta\\ \mathrm{Re}\,\beta\end{pmatrix}\end{aligned}$$

No, this concern would not invalidate the answer to (b.).

52.

$$\begin{aligned}I_{A+B} &= I_A + I_B + \\ &+ \sqrt{I_A I_B}\,\frac{\mathrm{Re}\,(\alpha\gamma^* + \beta\delta^*)}{\sqrt{(|\alpha|^2+|\beta|^2)(|\gamma|^2+|\delta|^2)}}\end{aligned}$$

53.
(a.)

$$J = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$$

(c.) No, the twisted nematic cell absorbs $\hat{y}$ polarized light, whereas the rotator rotates $\hat{y}$ polarized light into $\hat{x}$ polarized light.

56.
(a.) No, Pedrotti's Jones matrix completely absorbs LH circularly polarized light.
(b.) Light passes through a device consisting of a QWP with slow axis at 45° with respect to $\hat{x}$, followed by an $\hat{x}$ polarizer, followed by a QWP with slow axis at $-45°$. The Jones matrix for this combination is half of that in part (a.).

59.
(a.)

$$\begin{aligned}\begin{pmatrix}\mathcal{S}_0\\ \mathcal{S}_1\\ \mathcal{S}_2\\ \mathcal{S}_3\end{pmatrix} &\equiv \mathcal{S}_0 = \mathcal{S}_p + \mathcal{S}_n\\ &= \begin{pmatrix}\sqrt{\mathcal{S}_1^2+\mathcal{S}_2^2+\mathcal{S}_3^2}\\ \mathcal{S}_1\\ \mathcal{S}_2\\ \mathcal{S}_3\end{pmatrix} + \\ &+ \begin{pmatrix}\mathcal{S}_0 - \sqrt{\mathcal{S}_1^2+\mathcal{S}_2^2+\mathcal{S}_3^2}\\ 0\\ 0\\ 0\end{pmatrix}\end{aligned}$$

61.
(c.) There is no relationship between V and $\mathcal{V}$.

62.
(a.) Referring to the hint, $\delta = \frac{\pi}{6}$. Then, approximating Betelgeuse as two point sources separated by $\Delta\theta_{\mathrm{Betelgeuse}} = 0.047$ arc sec,

$$\text{slit sep} = \frac{2\delta}{\pi}\,\frac{\lambda}{\Delta\theta_{\mathrm{Betelgeuse}}} \approx 0.878\ \text{m}$$

(b.)

$$l_c = \frac{\lambda_0^2}{\Delta\lambda} \approx 19.5\,\lambda_0 \approx 11.7\,\mu\text{m}$$

64.
(b.)

$$\frac{n_3}{n_1} = \sqrt{\frac{n_T}{n_0}}$$

65.
(a.) For $n_L = 1.38$, $n_H = 2.35$, and $N = 6$,

$$T = 1 - \left(\frac{(\frac{n_L}{n_H})^{2N} - 1}{(\frac{n_L}{n_H})^{2N} + 1}\right)^2 \approx 0.0064$$

(b.) $T = 1$.

68. $I'/I_0 = 1/4$ because (see hint) integral (b.) is half of integral (a.).

69.
(d.)

$$\frac{I_N(\psi_x, \psi_y)}{I_1(0,0)} = N^2 \frac{\sin^2\left(\frac{Nkd}{2}\sin\psi_x\right)}{\left(N\sin\left(\frac{kd}{2}\sin\psi_x\right)\right)^2} \times$$
$$\times\,\mathrm{sinc}^2\left(\tfrac{ka}{2}\sin\psi_x\right)\mathrm{sinc}^2\left(\tfrac{kb}{2}\sin\psi_y\right)$$

70.
(a.)

$$\mathcal{R}(\psi_x) = \left(\frac{\sin(2kd\sin\psi_x)}{4\sin\left(\frac{kd}{2}\sin\psi_x\right)}\right)^2$$

(b.)

$$\mathcal{R}(\psi_x) = \tfrac{1}{4}\left(\cos\left(\tfrac{kd}{2}\sin\psi_x\right) + \cos\left(\tfrac{3kd}{2}\sin\psi_x\right)\right)^2$$

(c.)

$$\mathcal{R}(\psi_x) = \cos^2\left(\tfrac{kd}{2}\sin\psi_x\right)\cos^2(kd\sin\psi_x)$$

(d.) The three results are equivalent.

71.
This fuzzy thick slit is a convolution of a thick slit of width $\frac{a}{2}$ and a thick slit of width $\frac{3a}{2}$:

$$\mathcal{R}(\psi_x) = \mathrm{sinc}^2\left(\tfrac{ka}{4}\sin\psi_x\right)\mathrm{sinc}^2\left(\tfrac{3ka}{4}\sin\psi_x\right)$$

72.
(a.) At $\psi_x = 0$ the wave is pure x polarized.
(b.) The waves from the two slits are polarized orthogonally (RHCP *vs.* LHCP) and do not interfere:

$$\mathcal{R}(\psi_x) = \mathrm{sinc}^2\left(\tfrac{ka}{2}\sin\psi_x\right)$$
$$\psi_x(1^{\mathrm{st}}\ \mathrm{min}) = \arcsin\tfrac{2\pi}{ka}$$